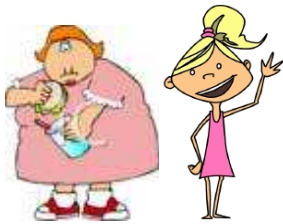


Brief look at the topics:

- Definition of ratio
- Change in ratios when numerator or denominator is multiplied or divided
- Equivalent ratios
- Comparing two ratios with same denominator
- Comparing two ratios with different denominators
- Brief notes on proportions.

Ratio

Knowing how to work with ratios comes very handy in our day to day life especially when working with different units of measurement. What really is a ratio? It's simply a comparison of two items, the relative size of two quantities expressed as a quotient of one divided by the other. Suppose we compare Rita's and Tina's weight, Rita is more obese and fatter than Tina.

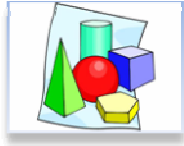


- ✚ Ratio tells how many times a quantity is greater or smaller than other

For example if a woman weighs 150 lbs and a child weighs 60 lbs, then woman's weight is $150/60$ times more than the child's weight. If we divide the numerator and denominator with their highest common factor, we get the ratio in the simplest form. So in this case common factor for 150 and 60 is 30, hence woman weighs is $5/2$ times than the child.

***Sometimes it is difficult to think of the highest common factor, so in that case divide numerator and denominator with the common factor that you can think of till there is no common factor left i.e. numerator and denominator are co-prime.**

- ✚ To compare two quantities, the physical units of measurement must be the same. For example you can't compare a box that weighs some tons to the one whose units are in kg. So first convert either tons to kg or kg to tons.



Ratio

- ✚ Ratio is a fraction, multiplying numerator and denominator with the same number keeps the ratio same!
- ✚ If numerator > denominator, ratio > 1 and when numerator < denominator, ratio < 1

How can it be expressed?

There are two ways, one is the old fashion, other, the new fashion.

Ex 1: Write the ration of 2 to 5

New way: as a fraction is, $\frac{2}{5}$

Old way: with a colon is, 2:5

Ex 2: Find the ratio of 7 inches to 3 feet

New way: as a fraction is, $\frac{7}{3}$

Old way, with a colon, is 7:3

Rate: Rate is a ratio between two different physical units such that it describes how the item in the numerator depends on the item in the denominator. It is basically described as 'value of the numerator for unit quantity of denominator.'

For example speed is described as distance traveled in unit time. So if you take the ratio of distance with time, you get the value of the distance traveled in one unit of time. But how? Say a car travels a distance of 50 km in 3 hour. How much distance it travels in 1 hour?

It is simple: Distance covered in 3 hours = 50 km

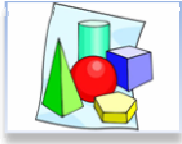
So distance covered in 1 hour = $(\frac{50}{3})$ km/hour

**** So in case of rate, please mention the units in the ratio.***

Problems having rates mostly have two ratios equal to each other and solve for an unknown number.

Comparing Ratios

- ❖ If denominator is same, then 2 ratios can be compared by comparing the numerator directly. Or if numerators are same, compare the denominators.



For Example:

$$\opl� \quad 4/7 \text{ [<, >]} 9/7$$

Since denominators are same , $4/7 < 9/7$

$$\opl� \quad 7/4 \text{ [<, >]} 7/2$$

Since numerators are same, so the ratio with smaller number in the denominator will be greater. Hence $7/4 < 7/2$

- ❖ If two ratios in simplest form are equal, then they are called equivalent ratios. For example, $4/12$ in simplest form = $1/3$. So $4/12$ is equivalent to $1/3$
- ❖ If the denominator and numerator of 2 ratios are different, then a typical long method of comparing the ratios is by finding the LCM of the denominator and making the denominators equal. Short cut method is the cross multiplication.

Fill the bank

$$5/3 <, >, = 7/9$$

Typical method: LCM of Denominators 3,9=9

So after making the denominators same we get $5 \cdot 3/3 \cdot 3$ and $7/9$

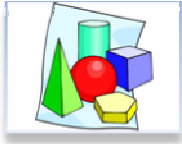
$$15/9 \text{ and } 7/9$$

since $15/9 > 7/9$ therefore $5/3 > 7/9$

Short cut method: By cross multiplication

$$5/3 \text{ and } 7/9$$

1. Multiply numerator of first with denominator of 2nd fraction i.e. $5 \times 9 = 45$. It represents the value for $5/3$
2. Multiply numerator of second with denominator of first i.e. $7 \times 3 = 21$. It



represents the value for $\frac{7}{9}$

3. comparing the 2 products we get $\frac{5}{3} > \frac{7}{9}$

How you got it? It is simple. Instead of calculating the LCM, the product of denominators is made the common denominator.

$$5 \cdot \frac{9}{3 \cdot 9} \text{ and } 7 \cdot \frac{3}{9 \cdot 3} = \frac{45}{27} \text{ and } \frac{21}{27}$$

Or

You can see in other way, whenever $\frac{a}{b} > \frac{c}{d}$ then cross multiplying on both the sides will give $ad > bc$.

Similarly $\frac{a}{b} < \frac{c}{d}$ gives $ad < bc$.

This will be true for any comparison operator in between two ratios like =, >, < etc.

Proportion:

As the name suggest proportion means being proportionate or balanced. Like the width of a door needs to be in proportion with the width of the room. For example for a room with a width of 12feet, the door should be of width 3 feet. Now if the architect decides to increase or decrease the width of the room, then in that case width of the door in his drawing should be changed accordingly. If suppose the width of the room is to be increased to 15 feet then what should be the width of the door?

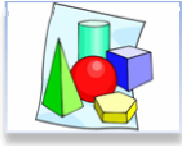
Tactic: Initial ratio of the width of the door to the width of the room= $\frac{3}{12}=\frac{1}{4}$

New width of the room=15 feet.

Since ratio of door width to room width (or room width to door width) should remain the same, we have $\frac{1}{4} = \frac{\text{(new width of the door)}}{15}$

=> width of the door=15 feet X $\frac{1}{4}$ =3.75 feet

Ratios are said to be in proportion when they are equal



Ratio

Examples: Cost per unit for same item will have same ratio for any number of those items

For Ex:

Are the ratios 1:2 and 3:4 equivalent?

Now, is $1/2 = 3/4$?

Cross multiply as explained earlier and compare LHS (left hand side) and RHS (right hand side)

$$\text{LHS} = 1 \times 4 = 4$$

$$\text{RHS} = 3 \times 2 = 6$$

$$\text{LHS} \neq \text{RHS}$$

Hence, the ratios are not equivalent.

If you are asked to find the ratio of two distances (for ex) in different units, first convert both the terms to same unit and then solve.

Ex 3: If 20 coats cost \$46, how much do 30 items costs?

$$\text{Cost/item} = \text{cost/item i.e. } 46/20 = x/30$$

Cross multiply,

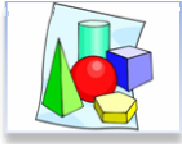
$$20x = 46(30), \text{ so } x = 46(30/20) = 69$$

Even with a calculator, this takes so much time, so we can use a trick, both 20 and 30 are multiples of 10.

20 cost \$46

10 coats cost \$23 (divide by 2)

30 coats cost $\$23 \times 3 = \69



Ratio

Ex 4: If 5 briggles equals 7 griggles, how many briggles = 11 griggles?

Briggles/griggles = Briggles/griggles, $5/7 = x/11$, $x = 11(5/7) = 55/7$,

Its quite simple, right?