Q 1) A daredevil plans to bungee jump from a balloon 65.0 m above a carnival midway. He will use a uniform elastic cord, tied to a harness around his body, to stop his fall at a point 10.0 m above the ground. Model his body as a particle and the cord as having negligible mass and obeying Hooke's law. In a preliminary test, hanging at rest from a 5.00 m length of the cord, he finds that his body weight stretches it by 1.50 m. He will drop from rest at the point where the top end of a longer section of the cord is attached to the stationary balloon.

(a) What length (in m) of cord should he use ?

(b) What maximum acceleration (m/s^2) will he experience?

Answer 1:

Let the mass of man is M. When he's suspended from 5 m long rope the weight must be balanced by the spring force, thus

$$M\ g = k_{l=5}\, 1.50$$
$$k_{l=5} = \frac{M\ g}{1.50} \qquad \text{Equation (1)}$$

Using conservation energy law, $W = \Delta K.E$,

$$W = W_{gravity} + W_{rope}$$
$$\Delta K.E = 0$$

Change in kinetic energy is zero since he's stopped at 10 m from ground, and initially also he jumps from rest.

$$W_{gravity} = M\ g\,(65-10)$$
$$W_{spring} = -\frac{1}{2} k_l x^2 \qquad \text{Equation (2)}$$

Also from the figure its can be seen that the length at maximum extension equals 55 m. Therefore,

$$55 = l + x \quad \text{Equation (3)}$$

From equation 1, 2 and 3 we have,

$$\frac{k_l}{k_{l=5}} = \frac{5}{l} = \frac{\frac{2\,M\,g\,(55)}{(55-l)^2}}{\frac{M\,g}{1.5}} = \frac{3\,(55)}{(55-l)^2}$$

In the above we've used the fact that spring constant is inversely proportional to the length of spring.

Solving the above equation we have,

$$(55-l)^2 = 33\,l$$
$$l^2 - 143\,l + 3025 = 0$$

$$l = \frac{143-\sqrt{(-143)^2 - 4\times 3025}}{2}$$
$$l \simeq 25.813\ m$$

b) Since it's a free fall the maximum acceleration experienced would be $g = 9.8\ m/s^2 downwards$

Q 2) A toy cannon uses a spring to project a 5.23 g soft rubber ball. The spring is originally compressed by 4.90 cm and has a force constant of 8.09 N/m. When the cannon is fired, the ball moves 15.9 cm through the horizontal barrel of the cannon, and there is a constant frictional force of 0.0326 N between the barrel and the ball.

(a) With what speed does the projectile leave the barrel of the cannon?
…………………..m/s

(b) At what point does the ball have maximum speed?
…………………..cm (from its original position).

(c)What is this maximum speed?
………………..m/s

Answer 2:

(a)Using energy conservation law, $W = \Delta K.E$ we have,

$$W = W_{spring} + W_{friction}$$

$$W_{spring} = \frac{1}{2} \times 8.09 \times (0.049)^2$$

$$W_{friction} = -0.0326 \times 0.159$$

$$\Delta K.E = \frac{1}{2} 0.00523 \times V^2$$

Friction acts opposite to direction of motion, thus work done by friction is negative. Work done by spring force is the negative of the change in its potential energy, since spring goes from compressed to uncompressed state the change in energy is negative thus work done is positive.

Solving the above equations we get, $V = 1.316 \;\; m/s$

b)
Maximum velocity will occur when all of the energy from spring has been transferred to projectile, this happens at x=4.9 cm.

c)
At x=4.9 cm, the work done by friction would be, $W_{friction} = -0.0326 \times 0.049$. Using the same equation as above, maximum velocity at x=4.9 cm would be $V_{x=4.9\ cm} = 1.7830 \;\; m/s$

Q 3)

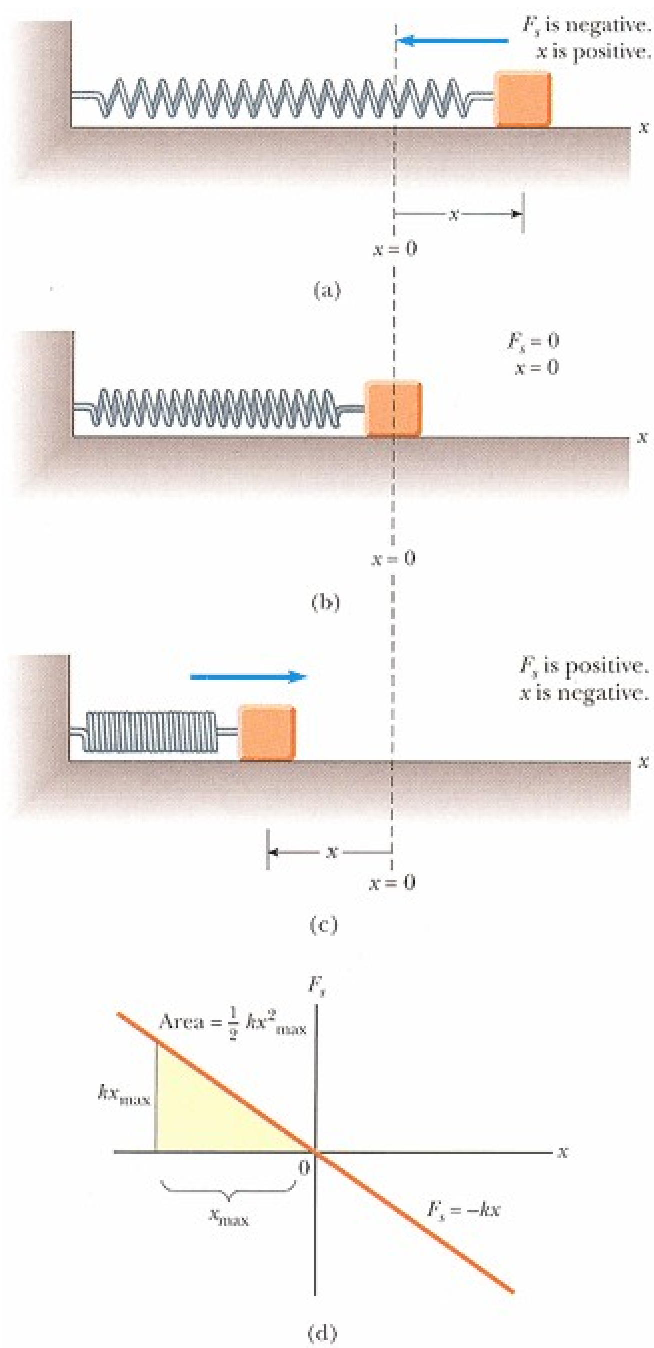
A 2.00 kg block is attached to a spring of force constant 525 N/m as in the figure below. The block is pulled 5.65 cm to the right of equilibrium and released from rest.

(a) Find the speed of the block as it passes through equilibrium if the horizontal surface is frictionless.

0.915 m/s

(b) Find the speed of the block as it passes through equilibrium (for the first time) if the coefficient of friction between block and surface is 0.350.

………m/s



F_s is negative.
x is positive.
x
x
x = 0
(a)
F_s = 0
x = 0
x
x = 0
(b)
F_s is positive.
x is negative.
x
x
x = 0
(c)
F_s
Area = $\frac{1}{2}kx^2_{max}$
kx_{max}
x
0
x_{max}
$F_s = -kx$
(d)

Answer 3:

(a)
Using conservation of energy law we have, $W = \Delta K.E$, where

$$W = W_{spring} + W_{friction}$$
$$\Delta K.E = \frac{1}{2}mV^2$$

$$W_{spring} = \frac{1}{2}\times 525\times(0.0565)^2$$

$$W_{friction} = -0.350\times 2\times 9.8\times 0.0565$$

Since friction acts opposite to direction of displacement the work done by it is negative. The displacement when it passes through equilibrium position is 0.0565 m.
Solving for V we' have

$$\frac{1}{2}\times 525\times(0.0565)^2 - 0.35\times 2\times 9.8\times 0.0565 = \frac{1}{2}\times 2\times V^2$$
$$V = 0.6711 \ m/s$$